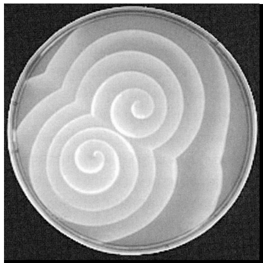

The 4th Law of Thermodynamics: power laws, coronal heating, reconnection & life

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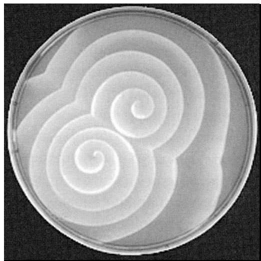
June 16, 2006

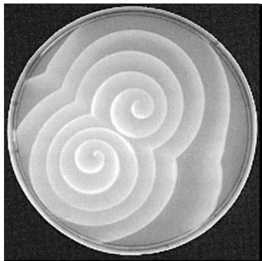


Abstract

The 2nd law of thermodynamics predicts that entropy (chaos) cannot diminish in equilibrium systems, but will increase until everything is maximally homogenized, cold and dead. Yet real life is full of counter-examples, from living organisms to the Sun's corona. The usual response from physicists is that these are all "non-equilibrium, open systems", but with no further physical insight into their properties. The field of "non-equilibrium thermodynamics" (NET) has recently gotten a big boost from the unlikely field of ecology, where it has proved very helpful in remote sensing applications. Simultaneously, mathematics has been developing tools that describe these NET systems, coupling the insights of fractal dimensions and non-random transport that produce non-equilibrium, power-law tails.

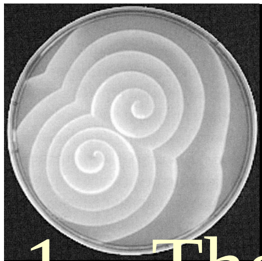
Ref: *Into the Cool*, Schneider & Sagan 2005.

Thermal Remote Sensing in Land Surface Processes, Quattrochi & Luvall 2006



Does Thermodynamics Matter?

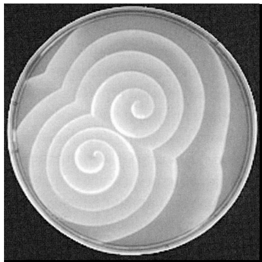
- Thermodynamics was the crowning achievement of 19th century physics, describing everything from nanoscale chemical reactions (chemical potential) to cosmoscale galactic evolution. To quote Eddington (1928):
  - If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations—then so much the worse for Maxwell's equations. If it is found to be contradicted by observation—well these experimentalists do bungle things sometimes. But if your theory is found to be against the 2nd law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.
- What about the Sun's corona? Power law tails? What about Life? Does the Anthropic principle really save (our) face? (Math's usual problem when physics appeals to "reality".)



How do Physicists save face?

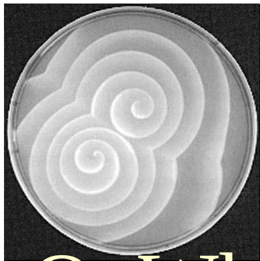
1. Thermodynamics only applies to closed, equilibrium systems. If we had more space, we could make this system closed.
2. If you we had more time, it would eventually come into equilibrium.
3. If we had more support, we could solve this problem.
4. Thermodynamics isn’t relevant today, we don’t do steam engines anymore, that’s engineering.
5. Entropy isn’t physics, it’s philosophy.
6. What a dumb question! Everyone knows that!

Physicist’s dark secret #17: We don’t really understand entropy.

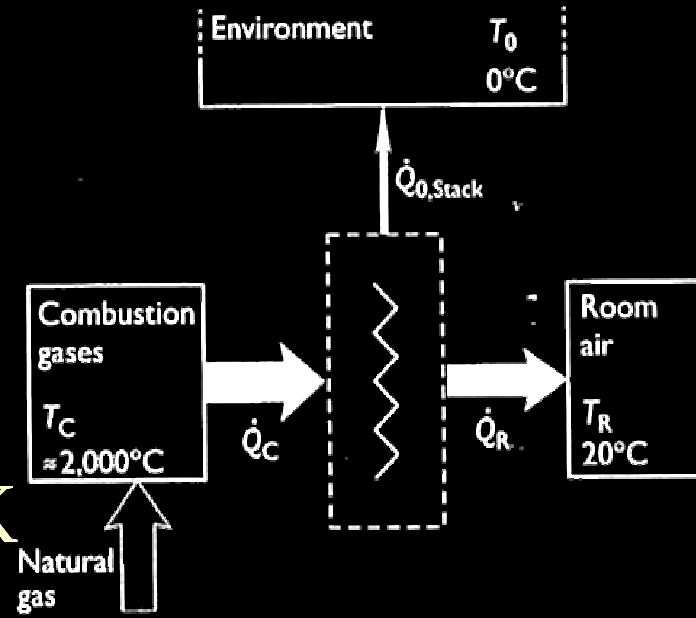


Really, Why Should I Care?

- Because the VP60 vision statement states we will "**Not be stovepiped**", clearly referencing the inadvisability of equilibrium thermodynamics.
- Because Non-Equilibrium Thermodynamics may be crucially important for scientific advance in "computationally challenging" problems, providing an additional constraint to otherwise intractable problems (global climate, coronal temperature, magnetic reconnection, plasma turbulence, astrophysical acceleration…)
- NET is truly more common than 19th century thermo. No? Let's have a test of your NET intuition.



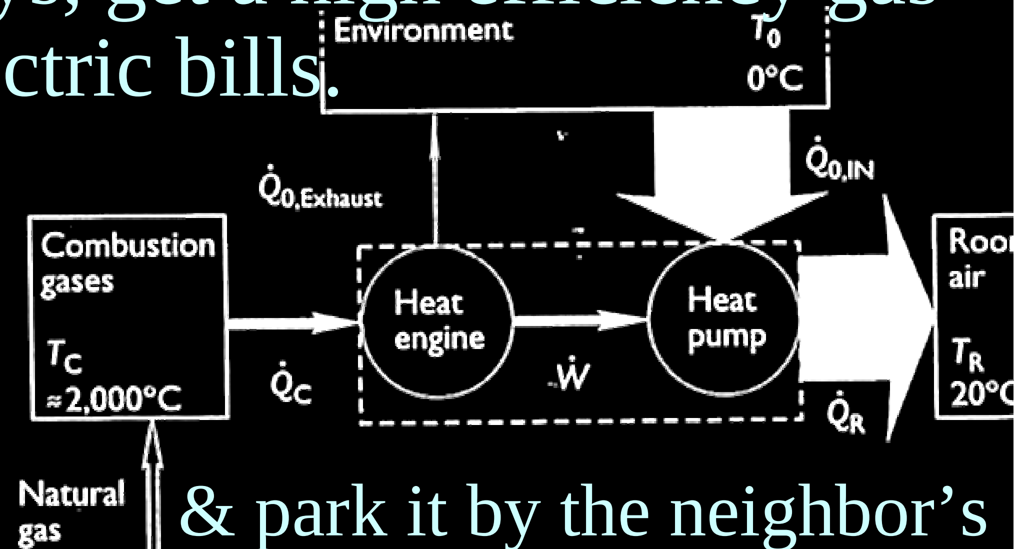
Exergy ≠ Energy



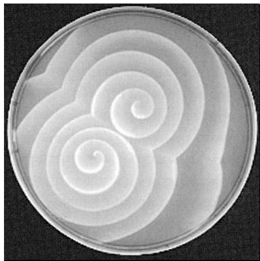
Q: What does it cost to heat your home if TVA charges $.05/MJ for electricity, and you have a 12,000 m^3 house you need to heat to 300K on a day when it is 275K outside? (SI units please!)

No, no, your neighbor says, get a high efficiency gas furnace and save $ on electric bills.

You fool, says the other neighbor, you could have bought a heat pump!



So you buy a Stirling engine & park it by the neighbor's

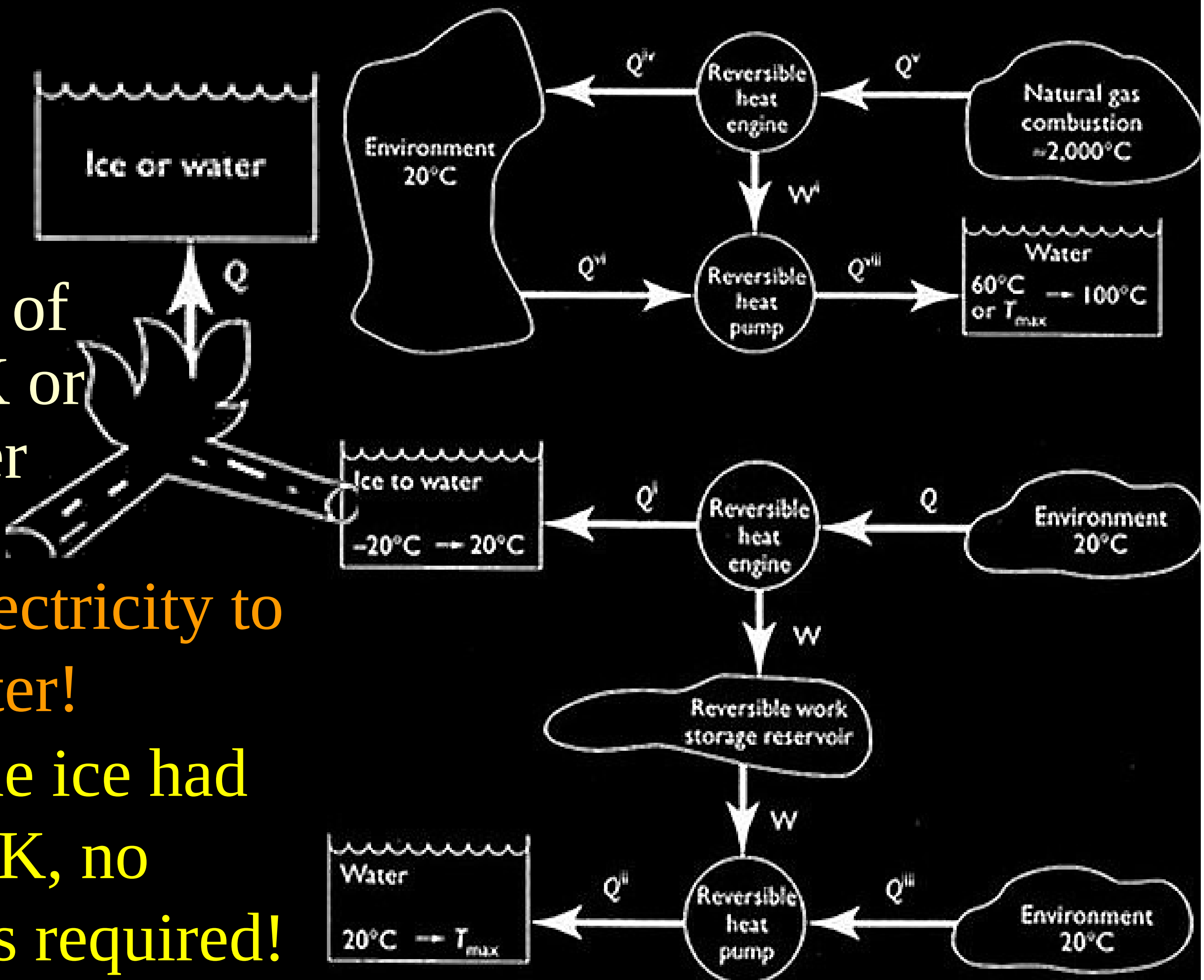


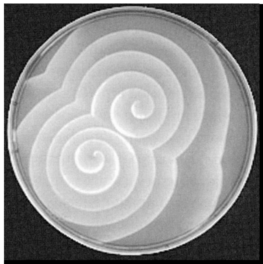
Test Question

What takes more electricity, boiling 1kg of ice @ 253K or 1kg of water @ 333K?

3X more electricity to boil the water!

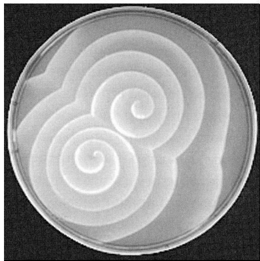
In fact, if the ice had been at 223K, no electricity is required!





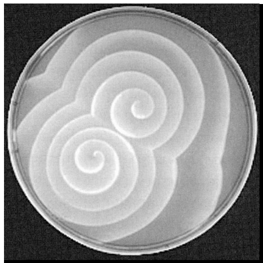
The Purpose of this talk

- Thermo is normally taught as a subset of energy conservation: engine efficiency, Carnot cycle, etc. In other words, as a scalar science.
- I hope to show that NET is about more than the scalar conservation laws, but also the vectors: the spatial gradients, the temporal gradients (flows).
- Just as Newton's force laws can be derived from Hamilton's energy principle, so NET is the dynamic equivalent of static (equilibrium) thermo. And like Euler-Lagrange, it too solves a superset of statics.



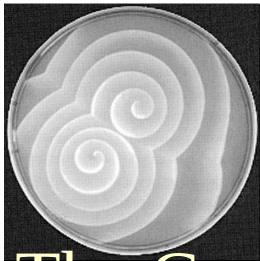
Outline

1. **The Scene:**
   Statistics and Thermodynamics
2. **The Crime:**
   Paradoxes of Nature
3. **The Clues:**
   Bénard & Ecology
4. **The Forensics:**
   Non-linear & Fractal
5. **The Conclusion:**
   Contingent Creation



1. The Scene

- Statistics
- Maxwell-Boltzmann Statistical Mechanics
- The Meaning of Entropy



Statistics

The Central Limit Theorem: The distribution of an average tends to be Normal, even when the distribution from which the average is computed is decidedly non-Normal, *except when the moments don't exist.* (Normal = Gaussian)

The average & width are rock-solid, empirical, invariants.

Paul Levy [1927] examined the exceptions.

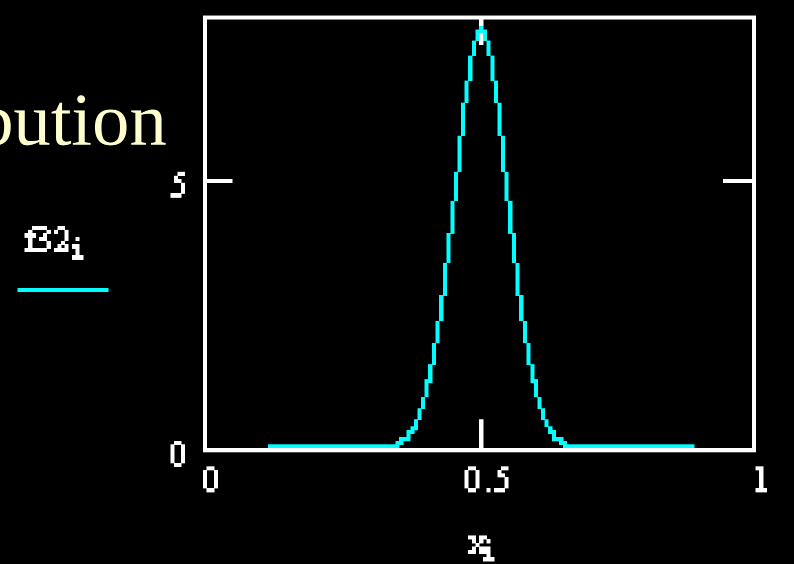
Variance: $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

< > requires the Probability Distribution Function, (PDF or P):

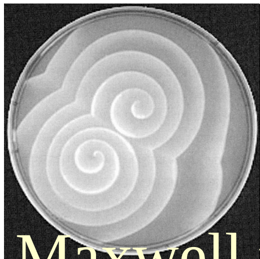
$$\langle x^n \rangle = \int dx\, x^n P(x)$$

- $P(x) \sim x^{-\mu}$
- *if* $\mu < 3$, $\langle x^2 \rangle = \infty$

  and $\sigma^2 \sim t^{1<\gamma<2}$



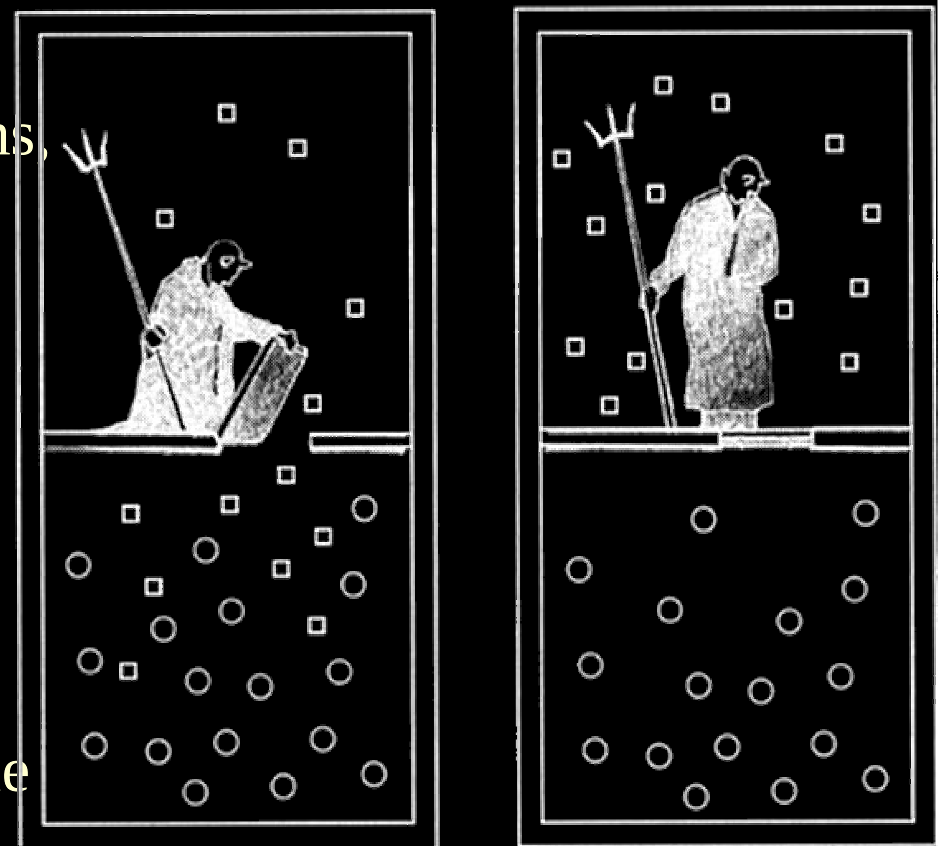
Distribution of Xbar when sample size is 32



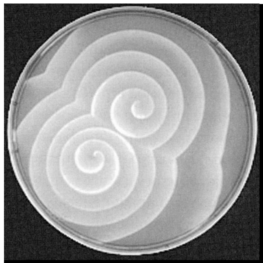
Maxwell-Boltzmann

Maxwell took the ancient Greek conjecture that matter is made of atoms, and starting deriving macroscopic quantities like pressure. Boltzmann applied even more sophisticated statistics and the whole field of thermodynamics fell apart like overbaked chicken.

But there were a few, just a few, annoying things about statistics. Why should every atom be indistinguishable in the statistical sense? Suppose there were a gnome (demon) who could separate fast from slow atoms (using radar, or ratchets etc.), wouldn't that destroy the 2nd moment? Wouldn't that allow heat to flow backwards?

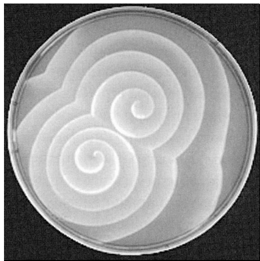


- Gnomes, demons and their virtual avatars, "information", are the opposite of entropy.



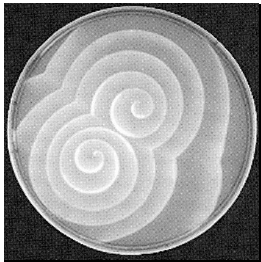
The Meaning of Entropy

- Macrostate (Clausius) S = (area/width) = Q/T
- Microstate (Boltzmann) S = log(possible)=k ln W
- Information (Shannon) S = Stirling approx=n ln n
- Optics (Young/Einstein) Coherence?
- Quantum (Jaynes) Negentropy?
- Astrophysics/Cosmology (Hawking)
- We are presently agreed (consensus science!) that Entropy and Information are inextricably entwined (Maxwell's demon, Quantum Eraser). This has implications for philosophy & cosmology.



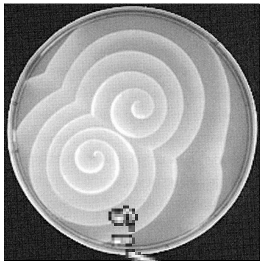
Summary of Statistics

- If the events are frequent enough (>50), independent of time (Markovian), independent of space (cross sections fall faster than $1/r^2$), independent of gradients in both time and space, THEN we can assume Gaussian statistics. We can assume normal diffusion, normal transport, normal heat flow, normal Epicurean materialism.
- Otherwise, we must rederive probabilities (Baysean), transport (Lévy flight), 2^{nd} moments (anomalous diffusion), entropy, and philosophy.

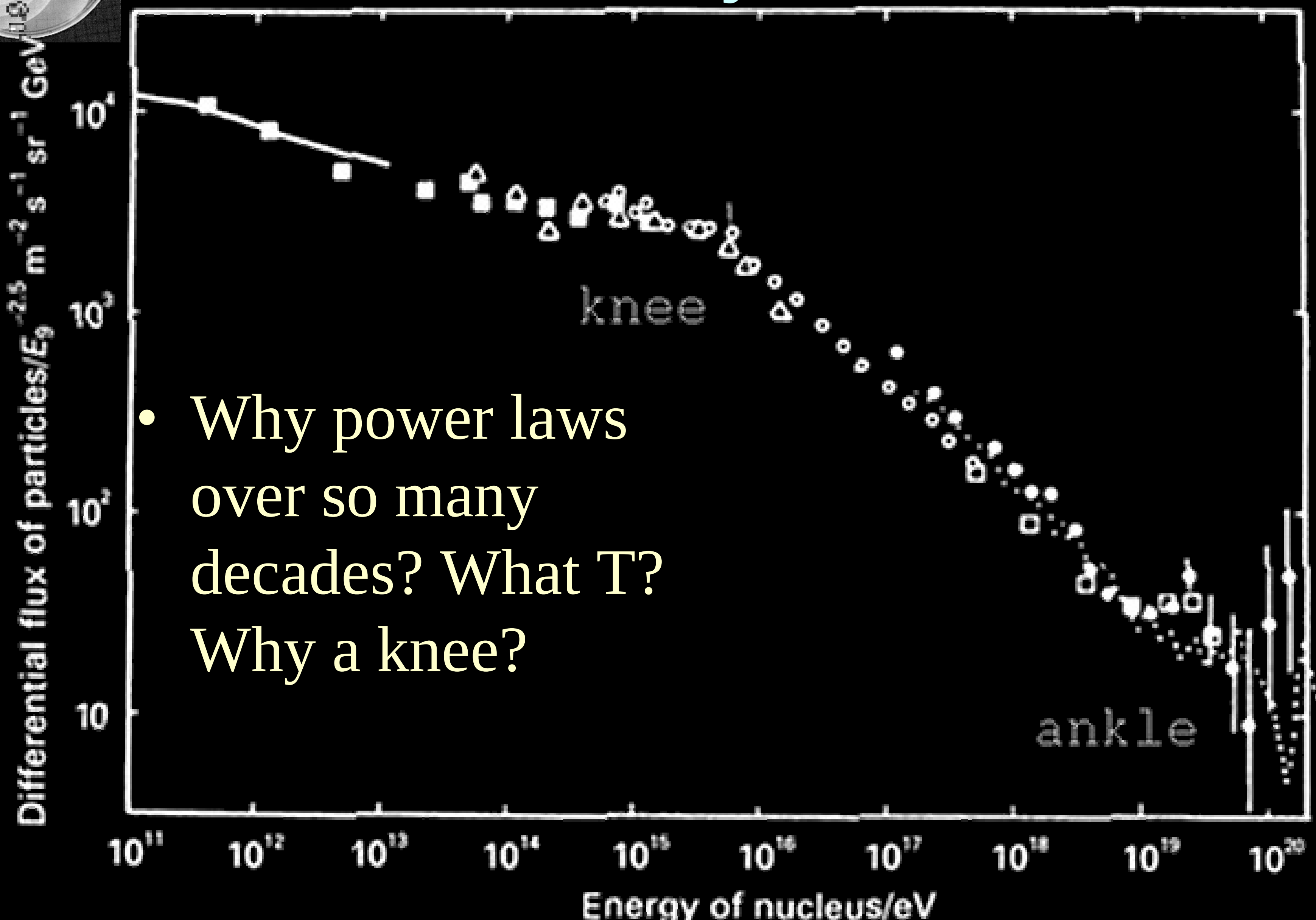


2. The Crime

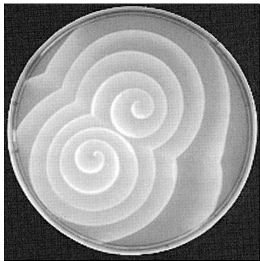
- Abnormal Acceleration:
  - –Cosmic Rays,
  - –Coronal Heating,
  - –Ring Current / Radiation Belts,
  - –Reconnection
- Orderly Chaos (Negentropy):
  - –Fractals, Galaxies, Life



Cosmic Rays > 100GeV

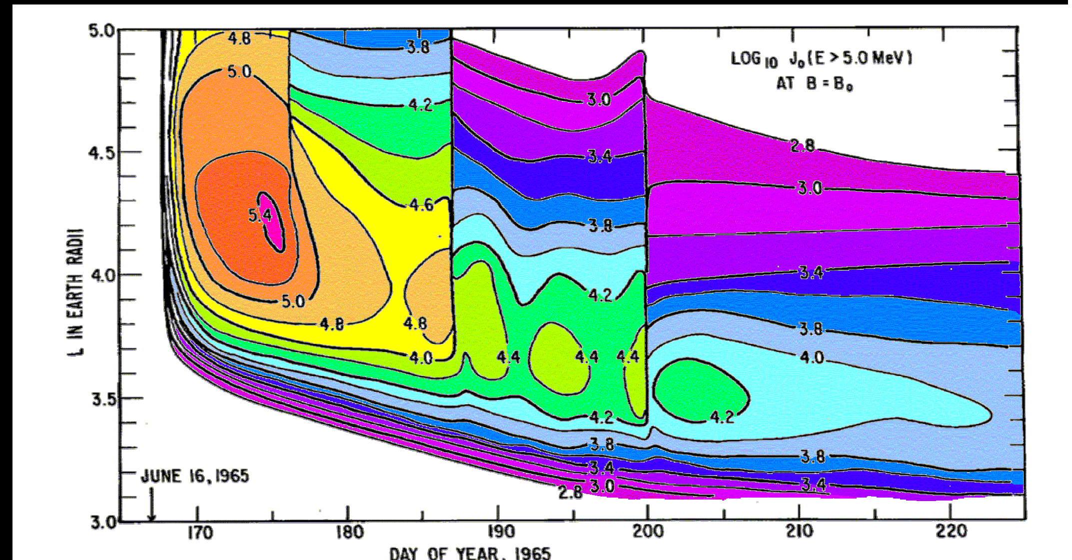


- Why power laws over so many decades? What T? Why a knee?

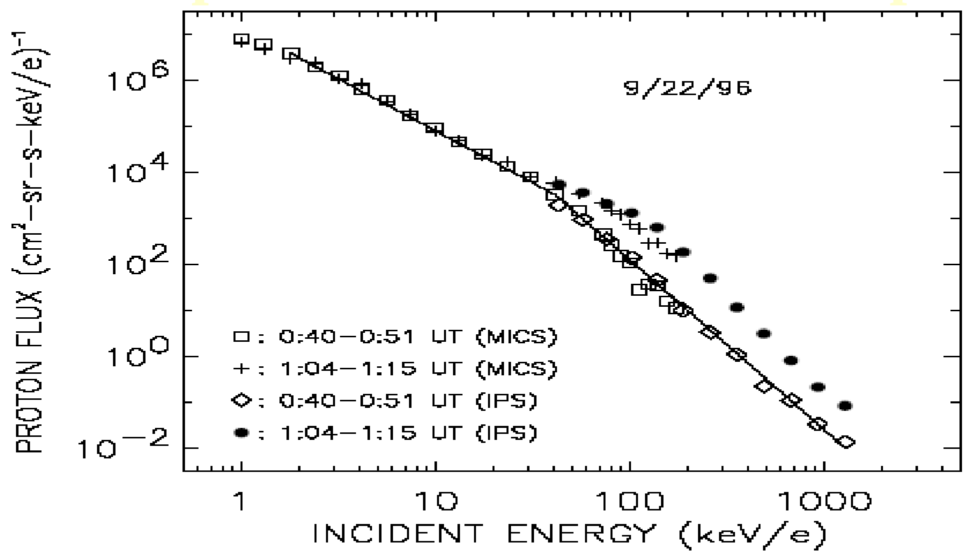


keV < Radiation Belts < 10 MeV

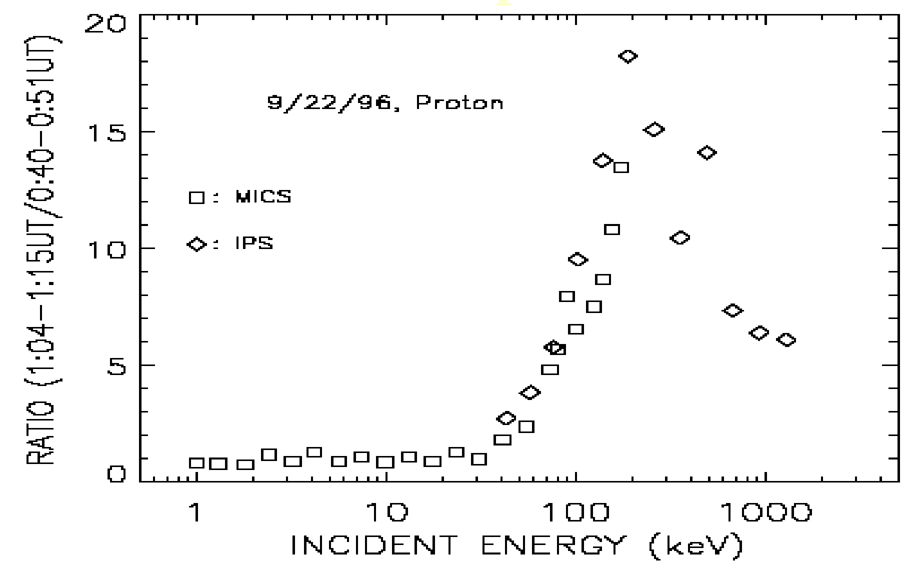
- T=5000 keV electrons in the radiation belts appear when T=1→2 keV solar wind.

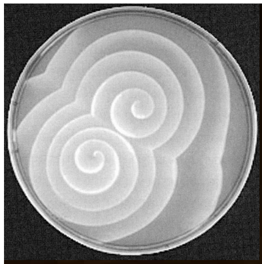


H+ Spectra at 2 times near cusp



Ratio of Spectra

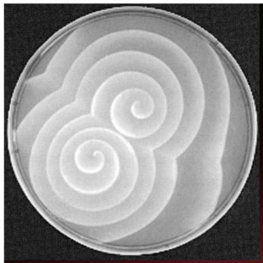




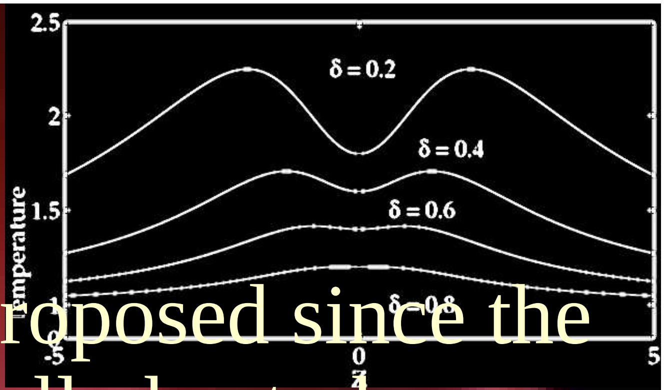
eV < Coronal Heating < keV

- Sun's visible surface = 5600K
- Sun's corona above = 2000000K
- How can heat/energy flow uphill?

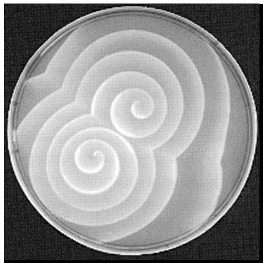
- If it's NET, what additional constraints can we adduce?



Magnetic Reconnection

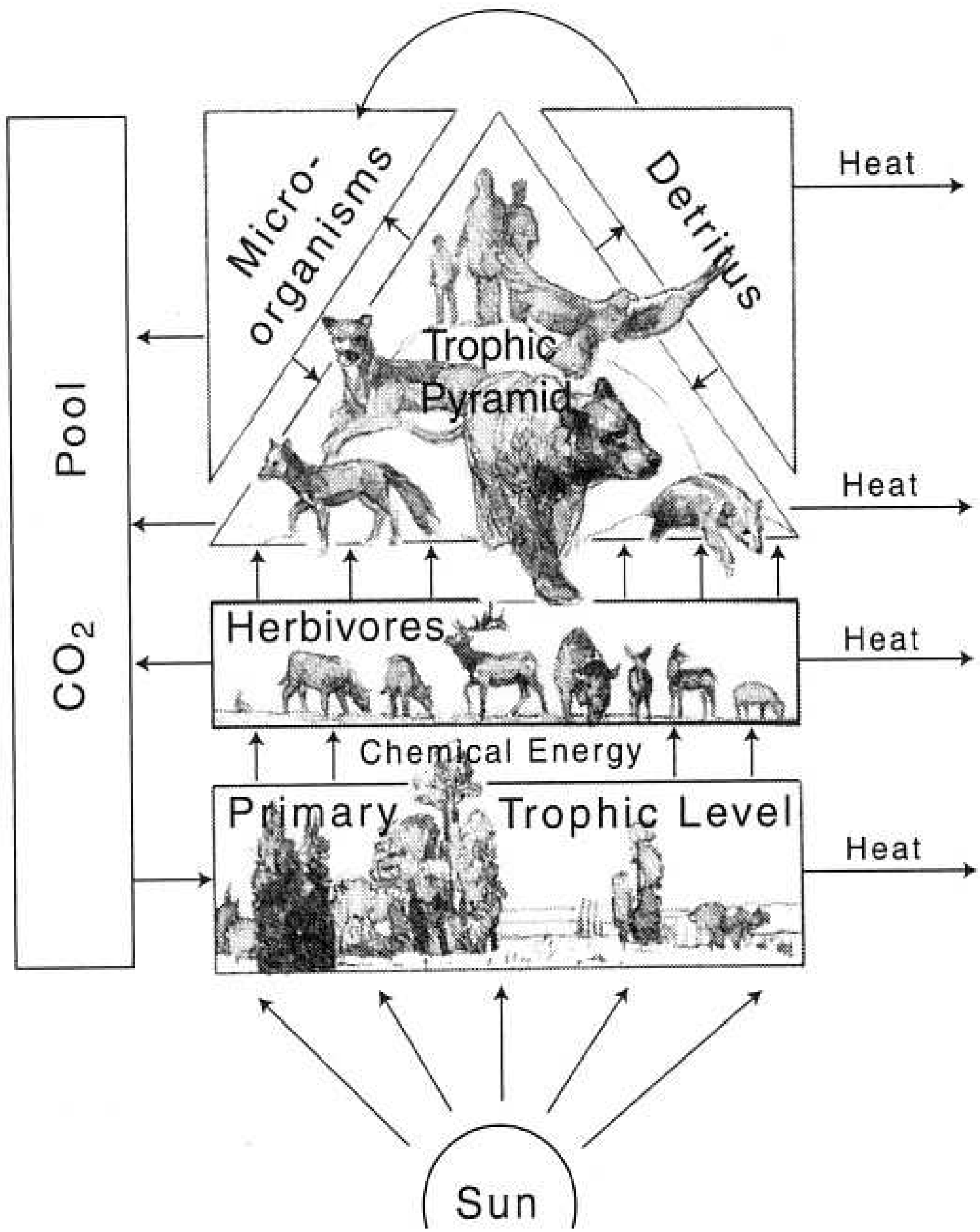


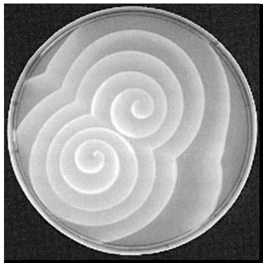
- Magnetic reconnection has been proposed since the early 1960's as a way to magnetically heat plasmas.
- The problem:
  - Neither the laboratory experiments, nor the analytic theory, nor the MHD/hybrid/PIC computer simulations show any substantial heating during the course of a reconnection. (Yoon 2006, Drake 2006)
  - The region in which this heating is supposed to occur in Nature, the anomalous diffusion region, keeps shrinking as our satellites & telescopes increase in resolution.
- Can magnetic reconnection be NET, and therefore not producing heat in the way we had thought?



Life

- Why does life seem to violate the 2nd law at all timescales?
  - Metabolism: Order maintained against the Chaos
  - Lifecycle: Birth → Death
  - Evolution: Speciation, complexity
- Is life an example of NET?



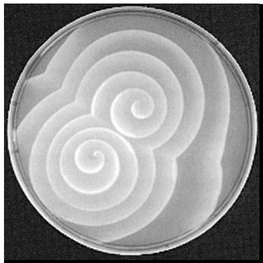


Summary of Paradoxes

- In space physics, just about every energy spectra we examine, cannot be characterized by a single temperature, as equilibrium thermodynamics requires for systems with so many particles.
- In all science subfields, there are examples of complexity increasing with time, in seeming violation of the 2^{nd} law.
- There are 2 possibilities:

1. The systems are NOT in equilibrium
2. The systems are in a NON-Gaussian equilibrium

- As it turns out, there may be deep reasons why the two solutions are equivalent



3. The Clues

- Bénard Convection Cells
- Ecology & Remote Sensing
- MEPP, Prigogine etc.

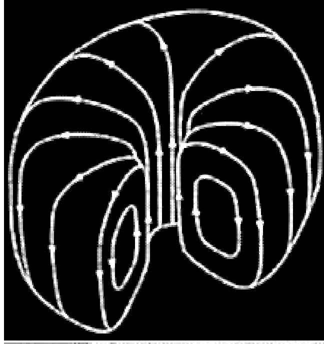
Bénard Convection

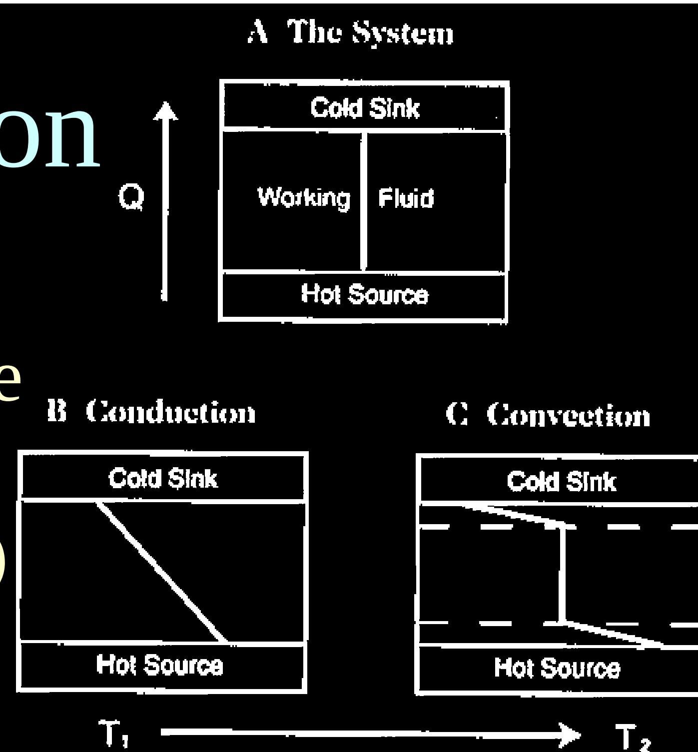
Rayleigh-Taylor (gradients)

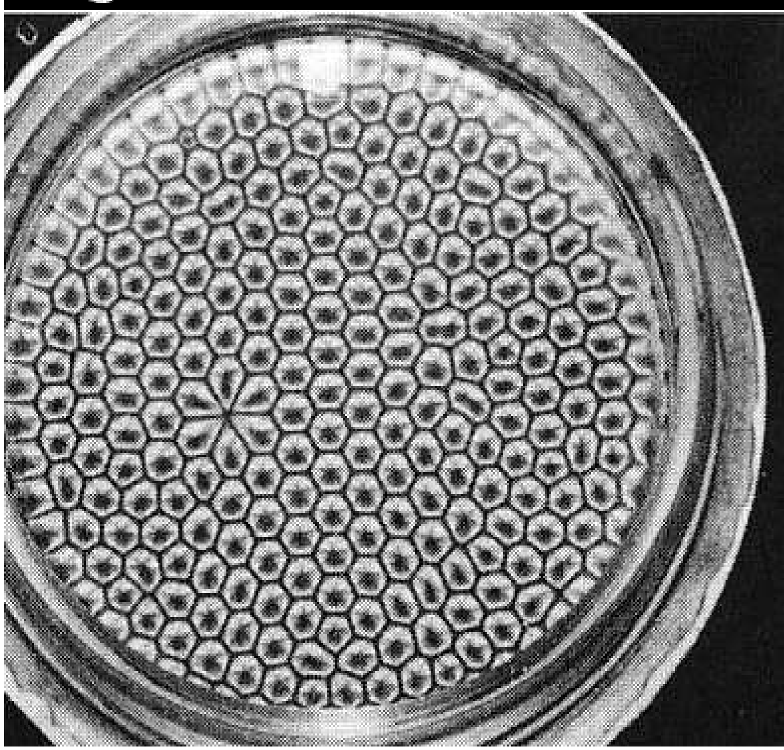
Lowest spatial mode unstable

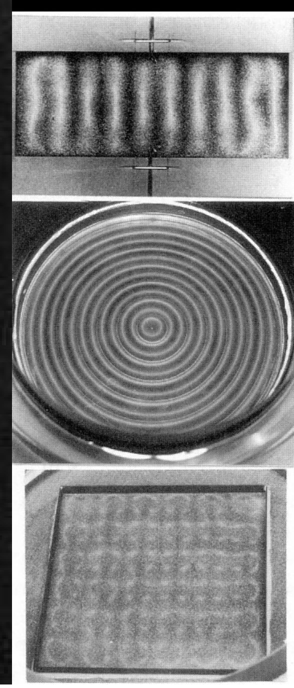
Boundary condition
determines form (not μΦ!)

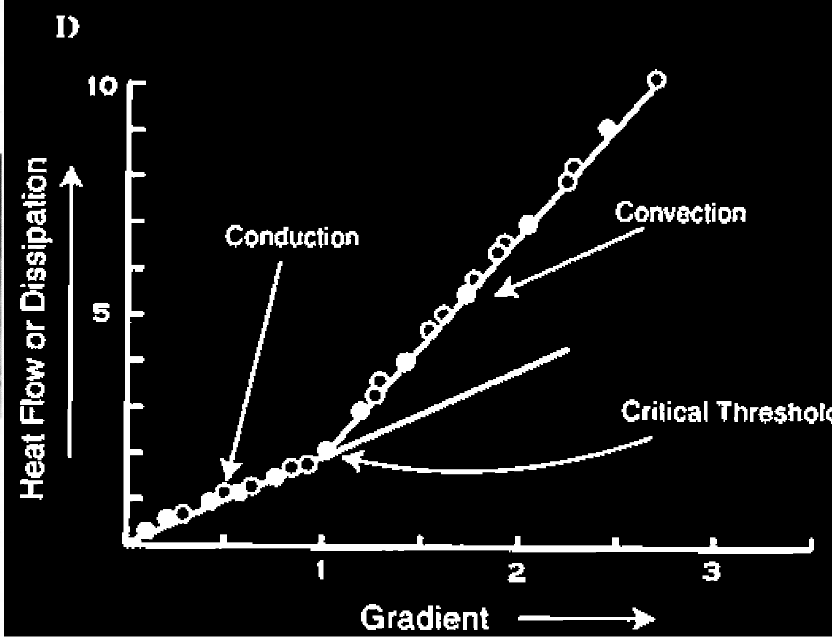
Matter cycles, energy flows

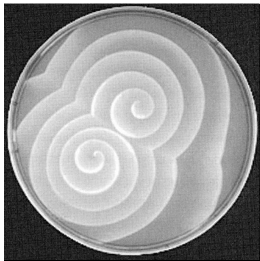




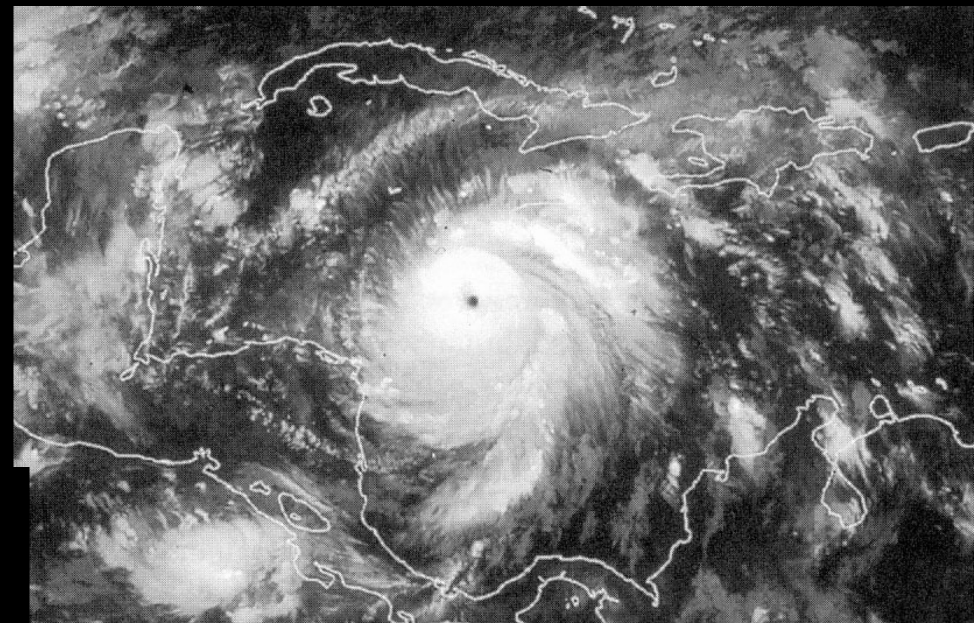




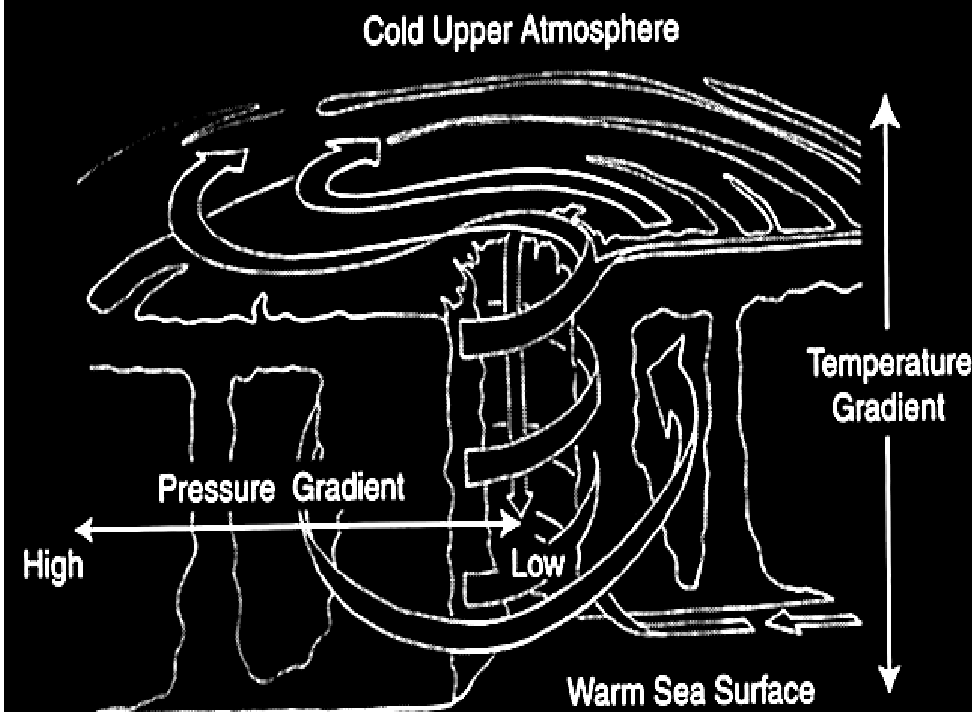


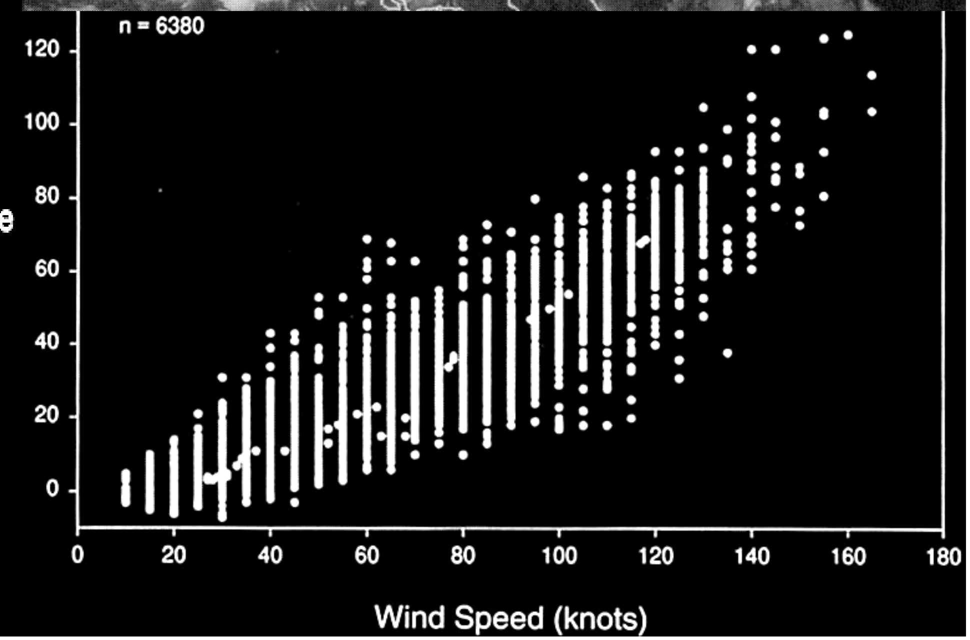


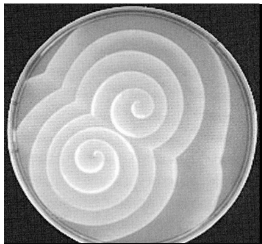
Hurricanes



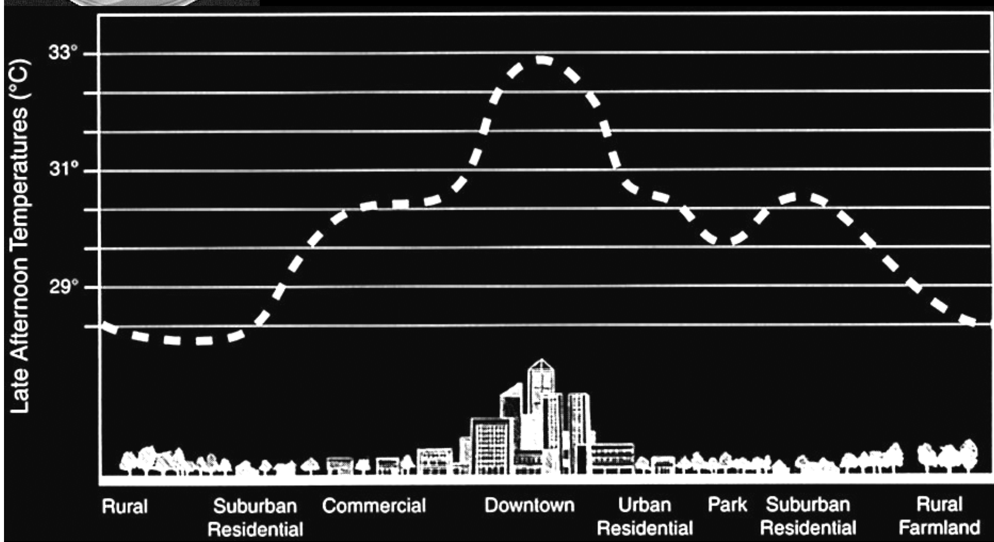
Stronger T gradients→
stronger P gradients→
higher wind speed→
faster dissipation

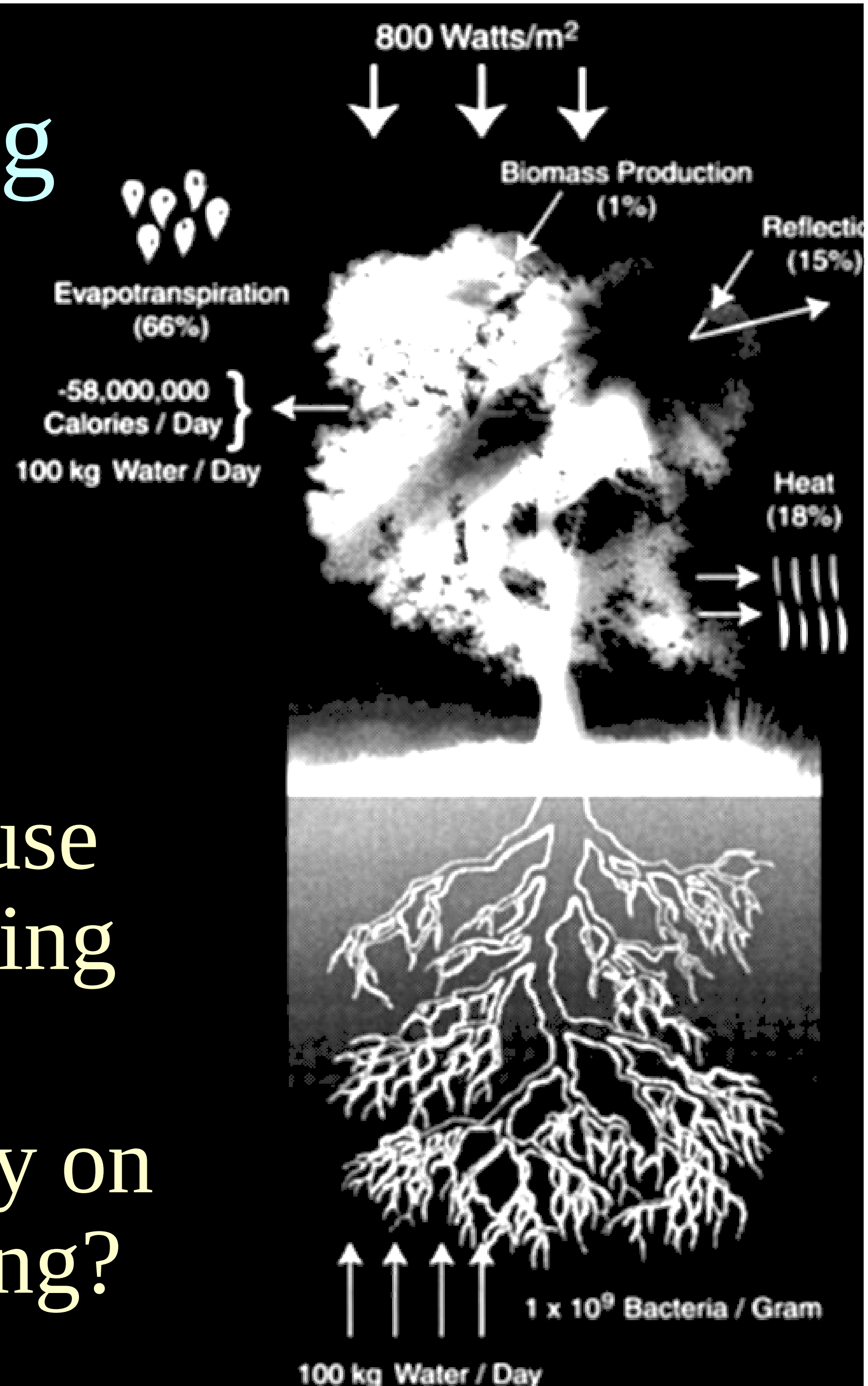






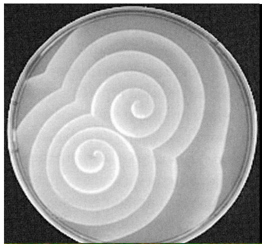
Remote Sensing





Why are cities hot? Because healthy vegetation is cooling itself off, unlike cities.

Why expend 2/3 of energy on cooling rather than growing?



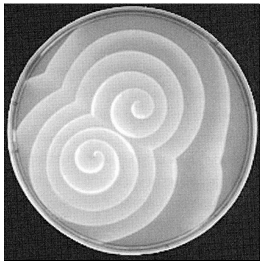
For exactly the same reason

- Gibbs Free Energy

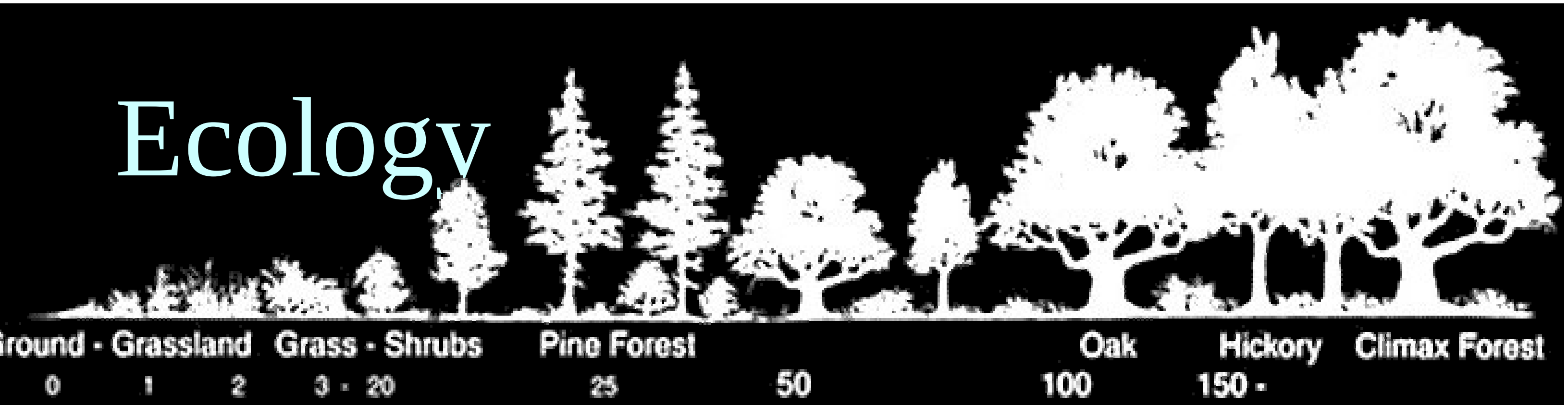
  G = H – TS

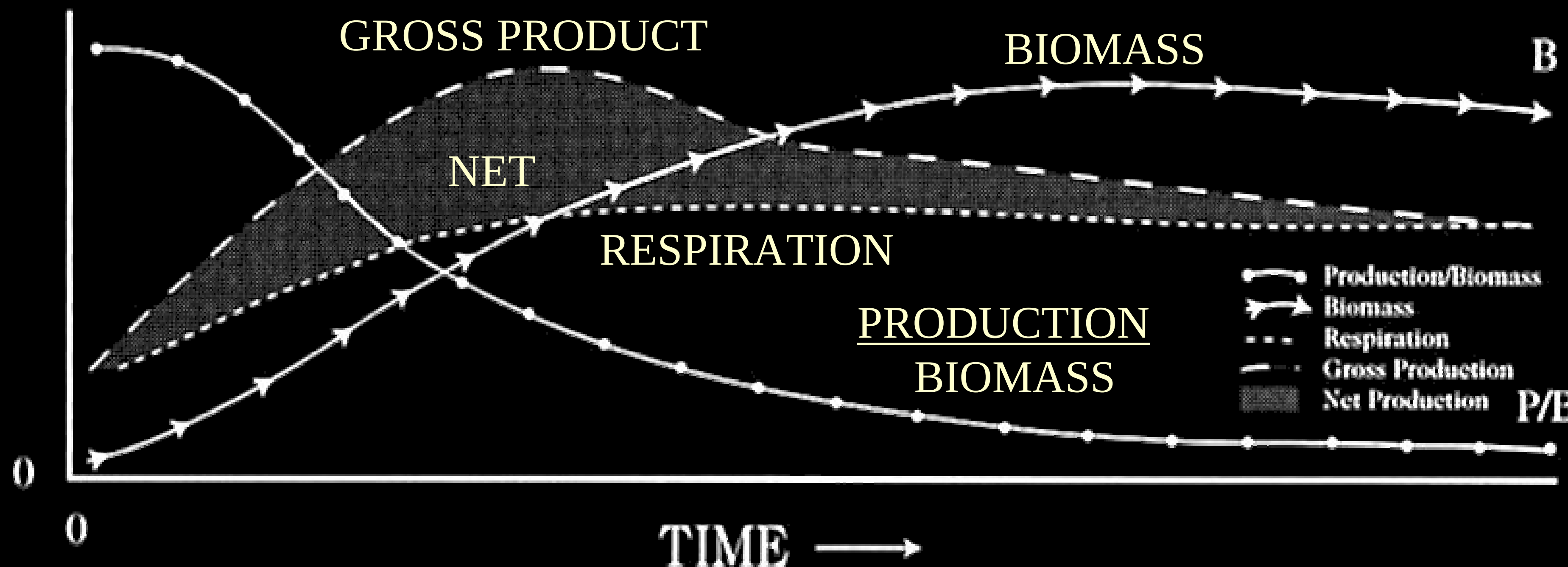
=“available energy” or Exergy.

So it is not only advisable but efficient to maximize G, by expending some energy to minimize T, =maximum gradient

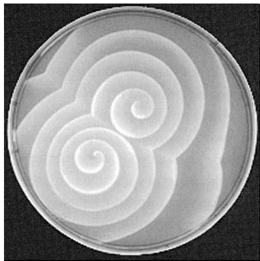


Ecology

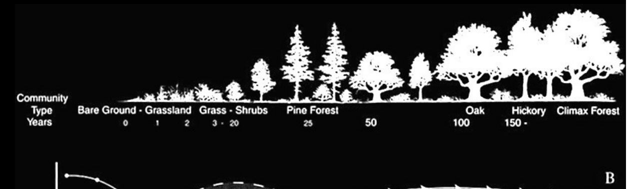




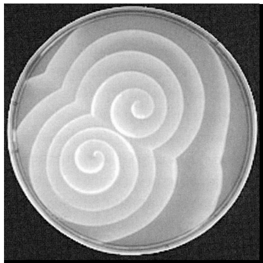
The more mature the forest, the more biomass, and the greater efficiency with which it is made. But for pure biomass, nothing beats grass. (Cows vs. paper mills. Kenaf)



Differences

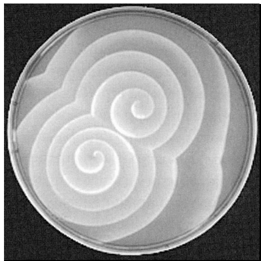


Juvenile, "Stressed"	Adult, "Unstressed"
High Fecundity, Growth	Low Fecundity/Development
Short Life Span	Long Life Span
Simple, Rapid	Complex, Slow
Few, Leaky cycles	Many, closed cycles
Near Thermodynamic Equil	Far Thermodynamic Equil
Low Free Energy, Exergy	High Free Energy, Exergy
High total S, Low S/kg	Low total S, High S/kg
Small Size, skewed neg. dist.	Large Size, unimodal dist.
Less complex, Low diversity	More complex, High diversity
Low system efficiency	High system efficiency



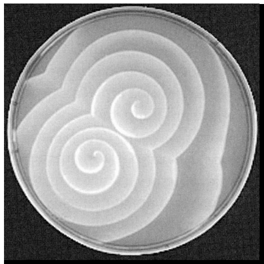
Maximum Entropy Production Principle (MEPP)

- A system not only moves toward greater entropy, (2^{nd} law), but on a path that maximizes the entropy production rate. (An application of the variational principle that derives Euler-Lagrange equations.)
- Prigogine's "Minimum Entropy Production Rule" is a restatement of the MEPP under additional constraints (but with unfortunate wording).
- If MEPP, then the 2^{nd} law can be derived as well.
- "Maximum exergy production", "Nature abhors a gradient", are all derivable from MEPP.
  - *Ref: "MEPP in physics, chemistry & biology" Martyushev & Seleznev, 2006 (Inst. Of Industrial Ecology, Ekaterinburg)*



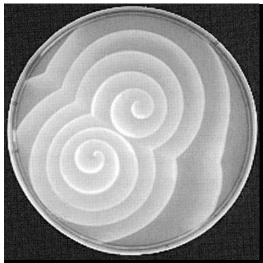
Summary of Clues

- When energy gradients exist in space or time, exergy, G=H-TS, is available. Systems that can extract the maximum exergy (long wavelength) grow at the expense of less efficient systems. If resources H&S are constant, then the system that minimizes T will have the more exergy available.
- So contrary to expectations, pushing more energy through a system does not necessarily raise T. In the case of trees, it reduces T! Energy flow should not be equated with temperature rise. Ditto for entropy.
- MEPP provides a quantitative description and constraint which can be applied to NET systems.



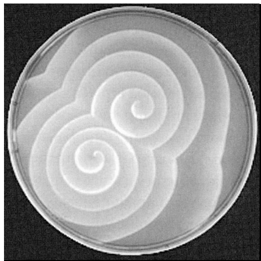
4. The Forensics

- Fermi’s Acceleration
- Weak Plasma Turbulence
- Mittag-Leffler Functions
- Fractional Calculus

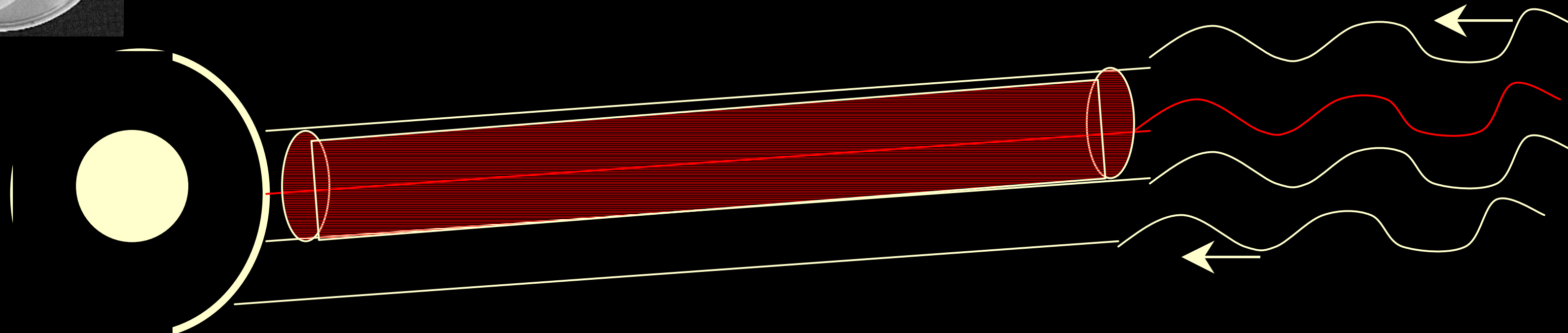


Forensics

- The power-law tails observed in all the abnormal accelerations in space, cannot arise from Gaussian statistics.
- They appear to come from NET systems.
- Can we derive them as the equilibrium of some process or physical law, and infer something about NET?

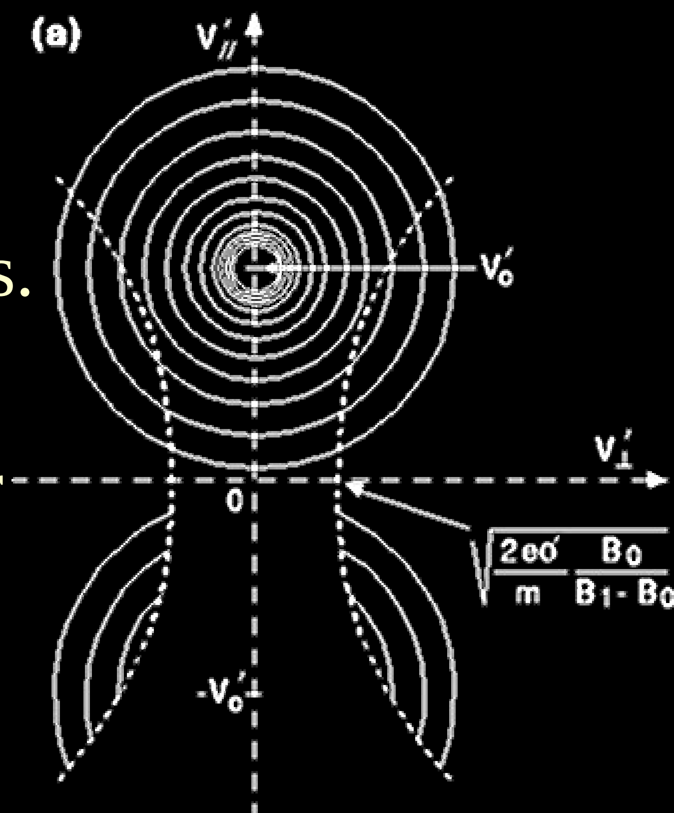


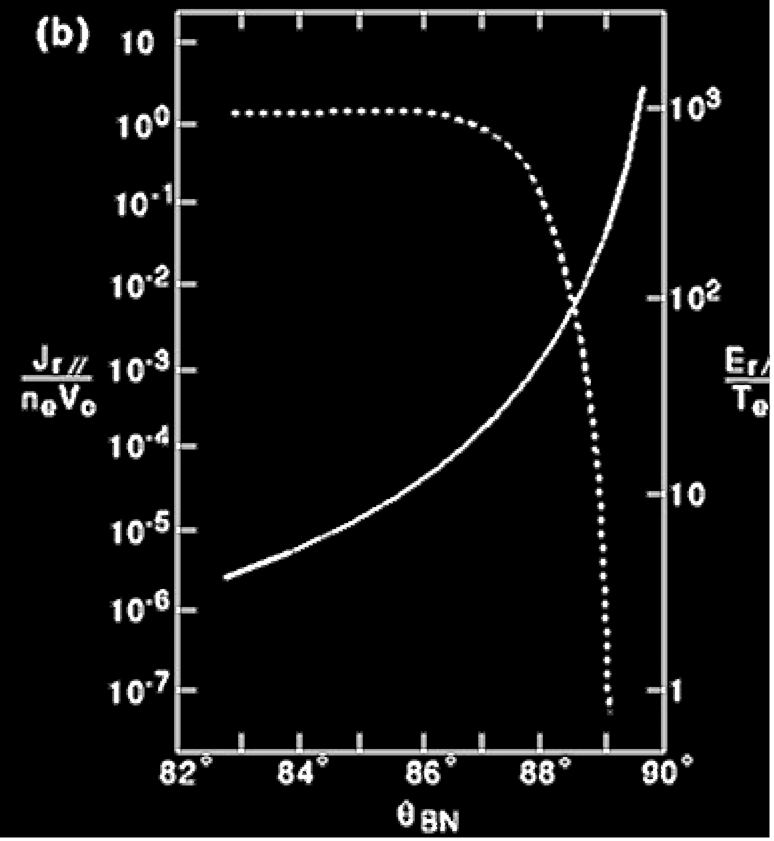
Fermi's Acceleration

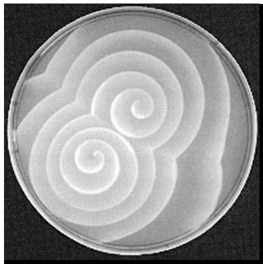


Fermi (1949) argued for acceleration between colliding walls. It's an astro-physicists dream, power law tails!

How? Gradients!

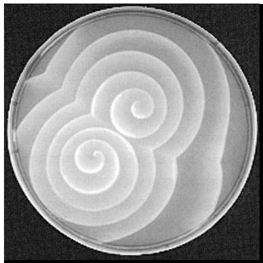




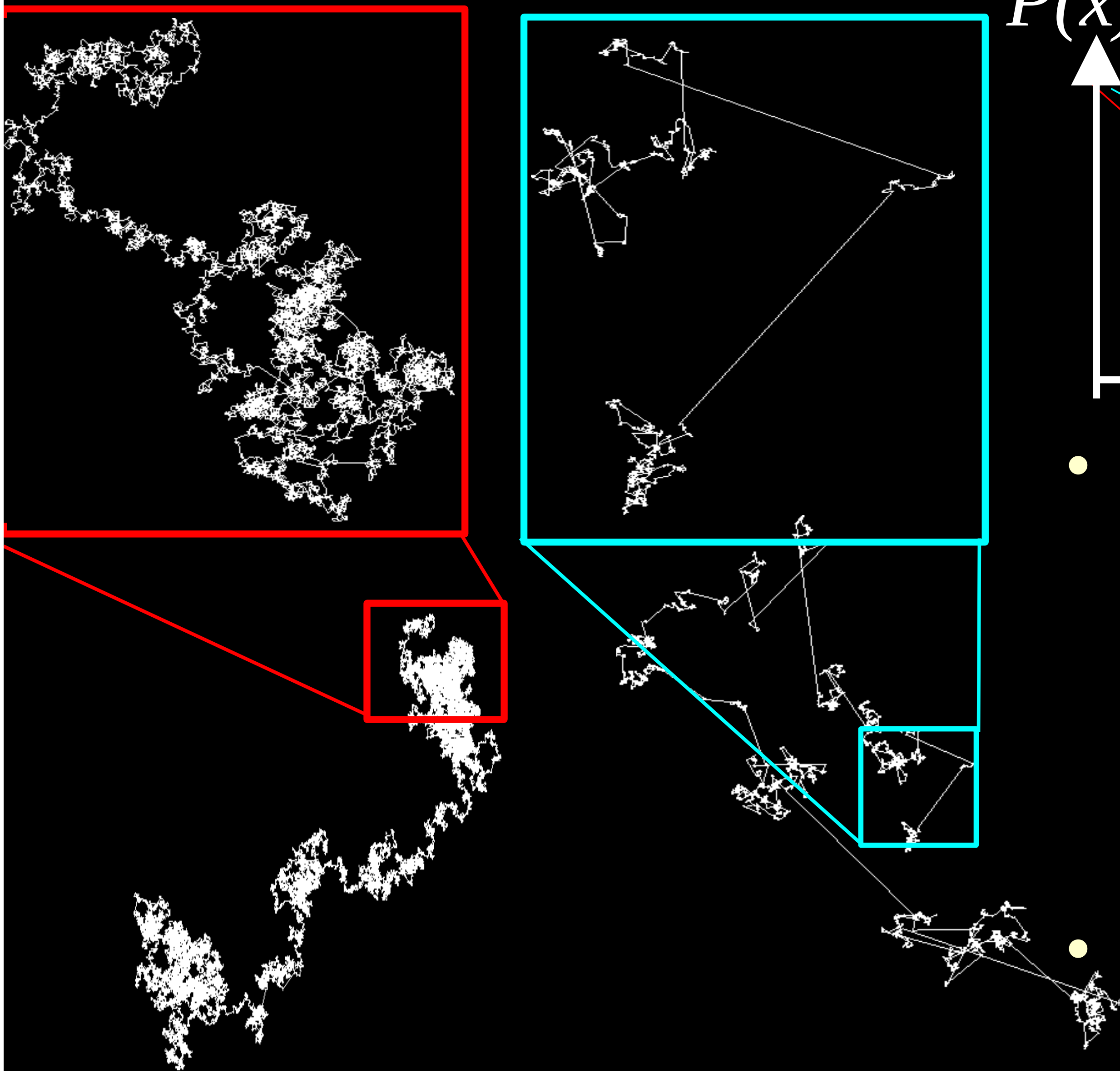


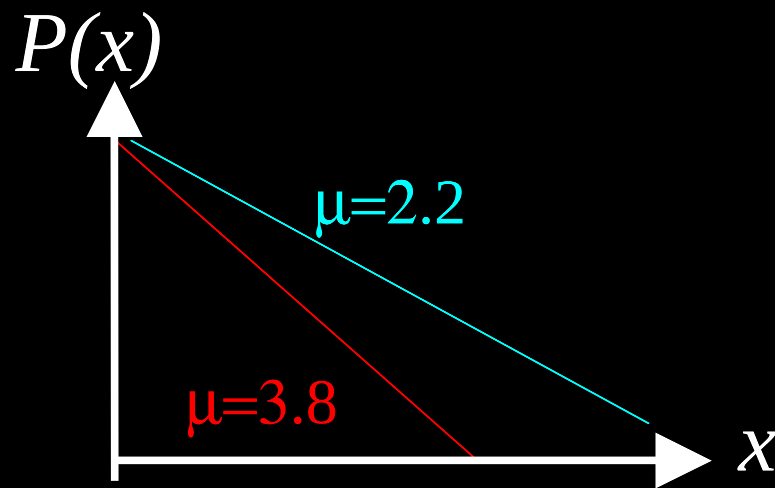
Weak Plasma Turbulence

- Non-magnetized beam-plasma interaction in the laboratory produces power-law tails on the beam energy. Plasma theorists addressed the challenge.
  - Turbulence theory developed in the 1960’s.
  - Quasi-linear theory (1970’s) didn’t get power-laws
  - Computer models (1980’s MHD) didn’t
  - Computer models (1990’s hybrid, PIC) didn’t
  - Fully non-linear theory (Yoon, *PRL*, 2004) did.
- Moral of the story: If the moments don’t exist (power-law tails), a bigger hammer won’t help.

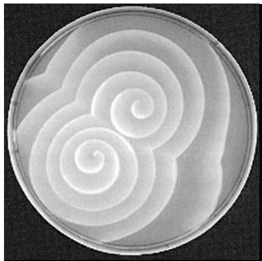


Diffusion vs Lévy Flight

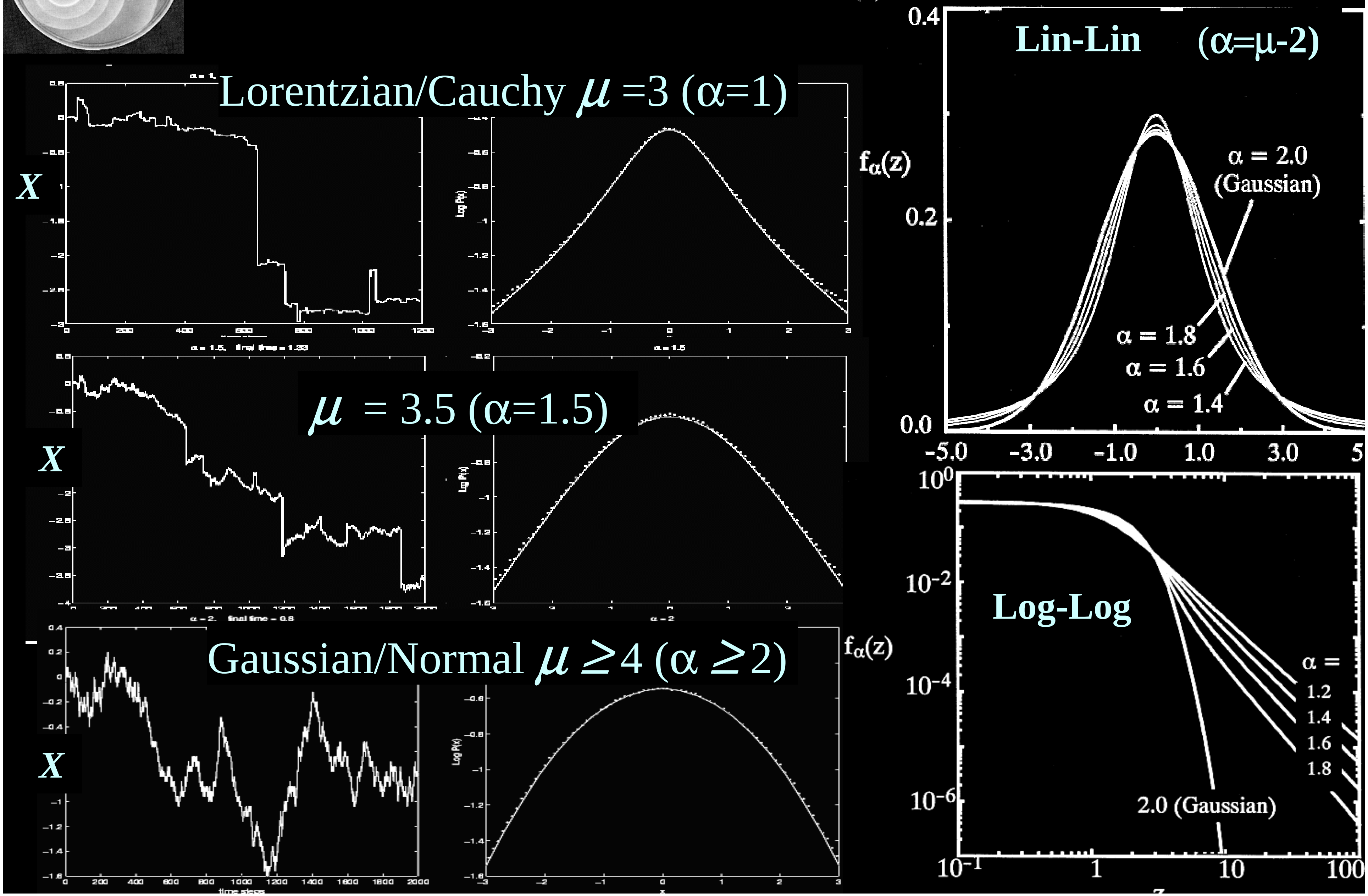


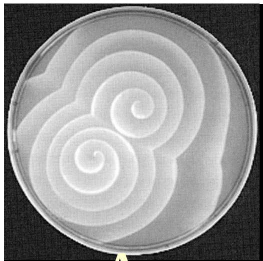


- A slight change in the PDF can change 2nd moment diffusion radically.
- Self-similar



Lévy-stable Distributions



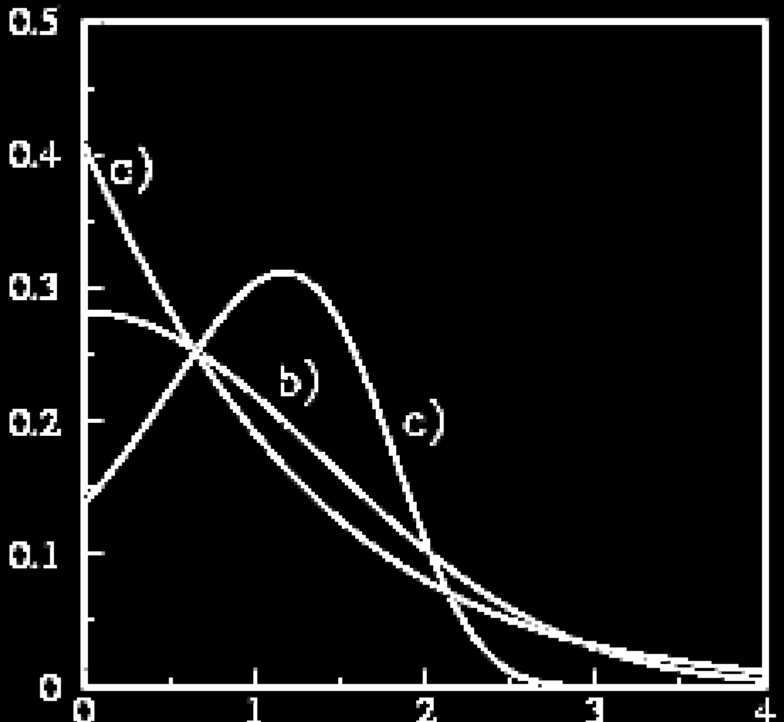


Mittag-Leffler Functions

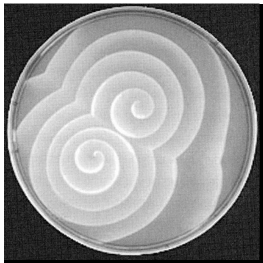
- A completely separate mathematical technique has been found to describe Lévy-stable distributions.

Time-fractional Diffusion Equation

- $d^n f / d^n t \;=\; D\, d^2 f / d^2 x$
- where D denotes positive constant with units of $L^2/T^{\mathbf{n}}$
- **n=2** wave equation; **n=**1 diffusion (heat) equation (Gauss)
- Anomalous Diffusion
- **a) n = 0** →Exponential decay
- **b) n** < 1 →slow subdiffusion
- c) n > 1 →fast superdiffusion

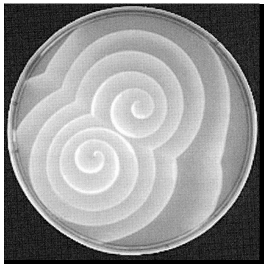


Solutions are Mittag-Leffler functions of order n, and *Lévy-stable pdf*



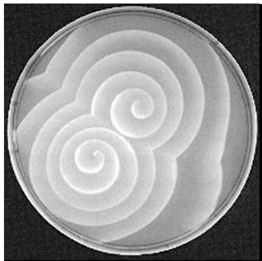
The Meaning of Fractional Transport

- The fractional derivative is integro-differential ➔ non-local.
- Structure introduces long-range interactions that destroy the premises of Central Limit Theorem. We can try to solve this with “normal” math, by dividing up the space in small pieces (ODE), then incorporate non-linearities to all orders. (Yoon). Note that MHD and PIC codes linearize!
- Conversely, we can integrate over all space, and treat the transport as a fractional derivative, which is just normal transport in a fractal dimension. Chandresekar’s Virial theorem demonstrated the advantages of this method.
- Therefore NET puts structure into the system, producing non-local effects, which are expressed as Lévy-stable dist.



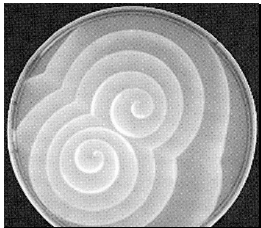
5. The Conclusion

- Math—Gaussian vs Bayesian Statistics (priors→ gradients).
- Acceleration—gradients!
- Order & Time's Arrow: gradients!
- Telos—Contingency: gradients?

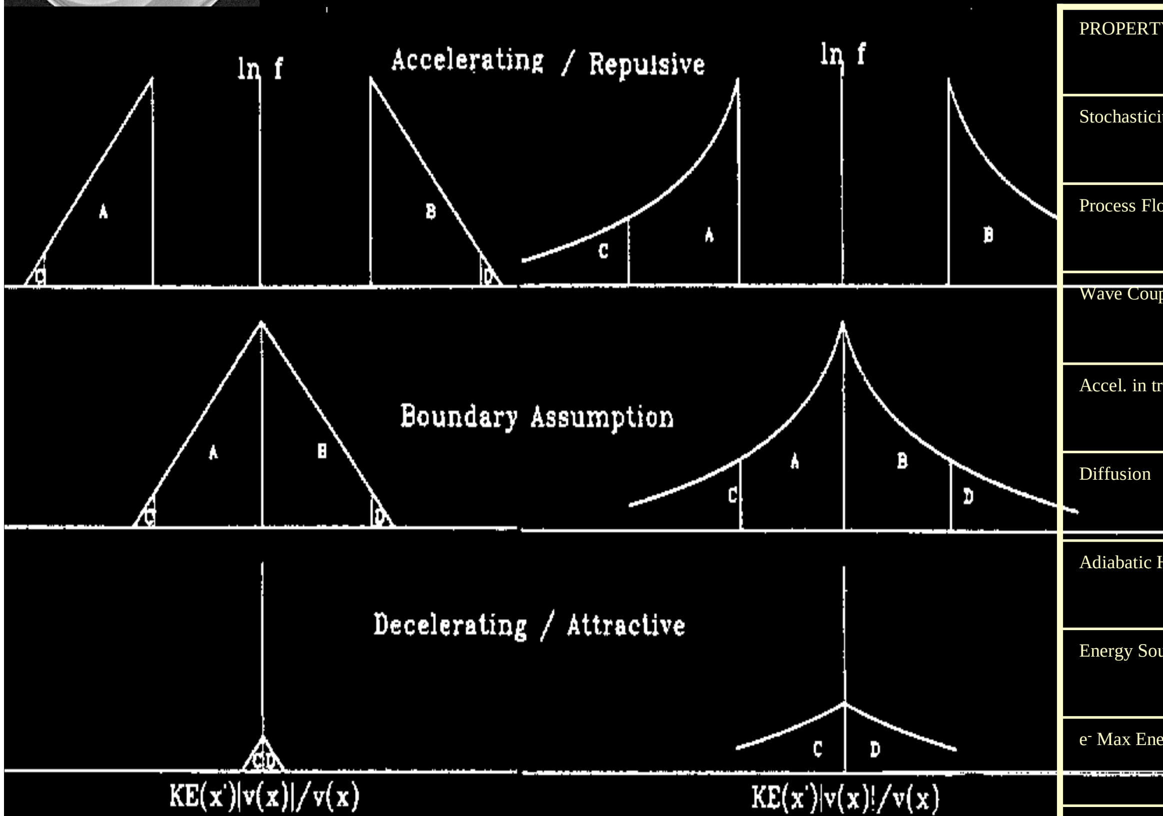


Math

- If you are analyzing a power-law tail problem, or suspect that you have a NET system, then throw away that statistics book, those F-tests and Chisqr fits. Check out the Bayesean statistics. (Sivia 1997)
- Since Gaussian statistics are a subset of Baysean, why wait until you have a NET problem? Do it now.

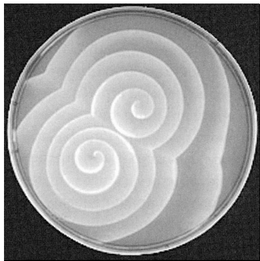


Acceleration



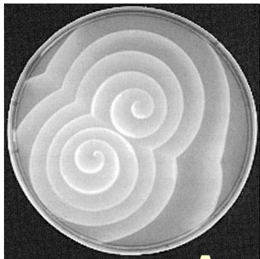
- Trying to evaluate competing mechanisms for acceleration? Use MEPP.

PROPERTY	DIPOLE	FERMI	QUADRUPOLE
Stochasticity	.001:1:1000 s	.001:>10^3:>10^4 s	0.1:1:10 s
Process Flow	rim>ctr>blocked	end>side>diffus	ctr>rim>open
Wave Coupling	hi E weak	all E same	hi E best
Accel. in trap	Traps	Detraps	Trap/Release
Diffusion	Essential	Helpful	Neutral
Adiabatic Heat	2D pancake	1D cigar	2D pancake
Energy Source	SW compress	SW Alfven	SW+internal
e^- Max Energy	900MeV@10Re	1.8 MeV@.1Re	280 MeV@3Re
e^- Min Energy	45 keV	2.5 keV	30 keV
Trap Volume	10^{24} m^3	10^{20} m^3	10^{22} m^3
Trap Lifetime	> 10^{13}s	10^4s	10^9:10^5s
Accel. Time	> 300,000s	8,000s	25,000s
Trap Power	< $5x10^8$W	10^6W	$5x10^7$W



Time's Arrow

- Having trouble with figuring out whether time is going forward or backward?
  - Elevator shoes, burgundy stripes and tube tops are back?
  - We are going to the Moon with what technology?
- Then you need the MEPP.



Telos

- And the ultimate question of all, in the beginning, was the Big Bang a high or low entropy event?
- Hot dense fireballs ought to have really high entropy. So where did all this structure in the Universe come from?
- Gravity gradients.
- But if gradients are negentropy, then the Universe must be packed with information.
- And we're still unpacking.
- With MEPP.

Soli Deo Gloria